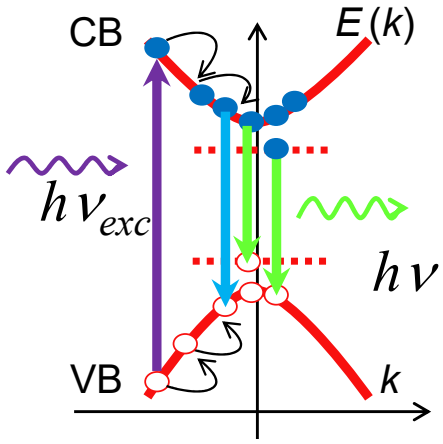
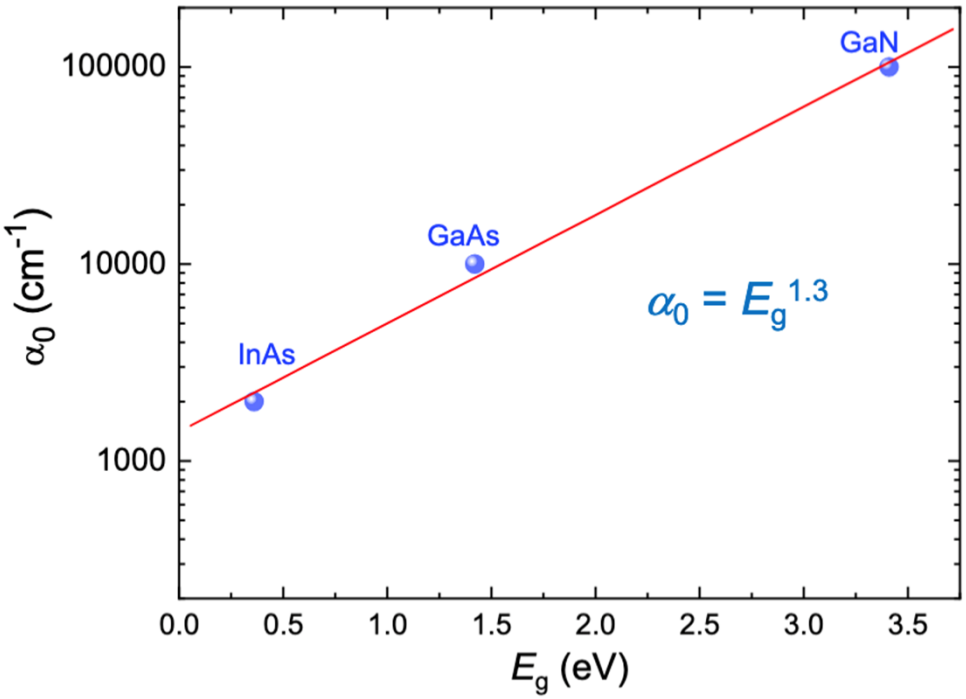
Lecture 14 – 18/12/2024

- Oscillator strength of excitons in semiconductors
- Electron-photon interaction beyond the semi-classical picture
- Spontaneous emission rate and bimolecular recombination coefficient
- Basic insights into near band edge photoluminescence transitions



Summary Lecture 13 - Absorption in semiconductors

$$\alpha_0(\omega) = \frac{q^2 x_{vc}^2 \omega}{4\pi\varepsilon_0 \hbar n_{op} c}\left(\frac{2m_r}{\hbar}\right)^{3/2}\sqrt{\omega - E_g/\hbar}$$



How does the absorption explicitly depend on the bandgap?

→ Missing dependence of n_{op}

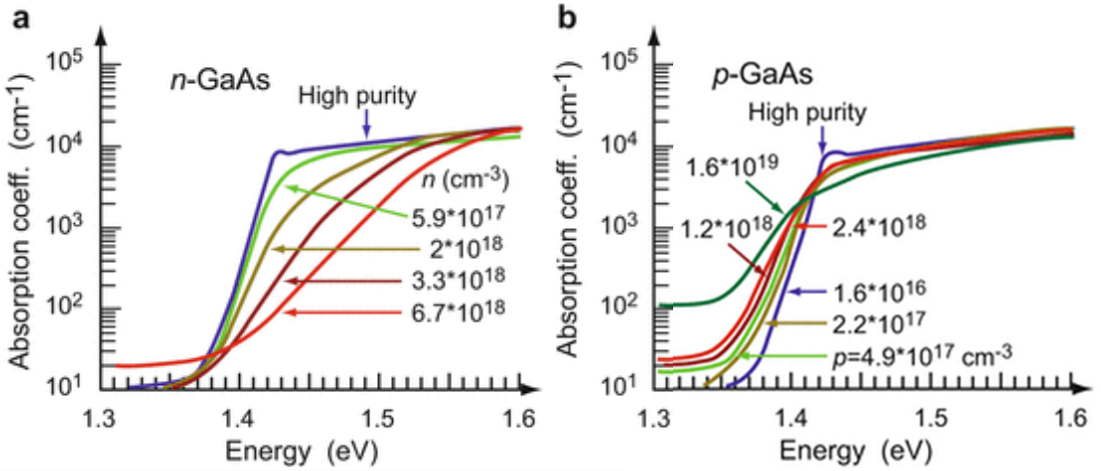
→ $$P(t) = qx(t)N = \frac{q^2 N E(t)}{\left[m_0\left(\omega_0^2 - \omega^2\right)\right]}$$ ✚ $\varepsilon = \mathbf{D/E} = \varepsilon_0 + \mathbf{P/E}$

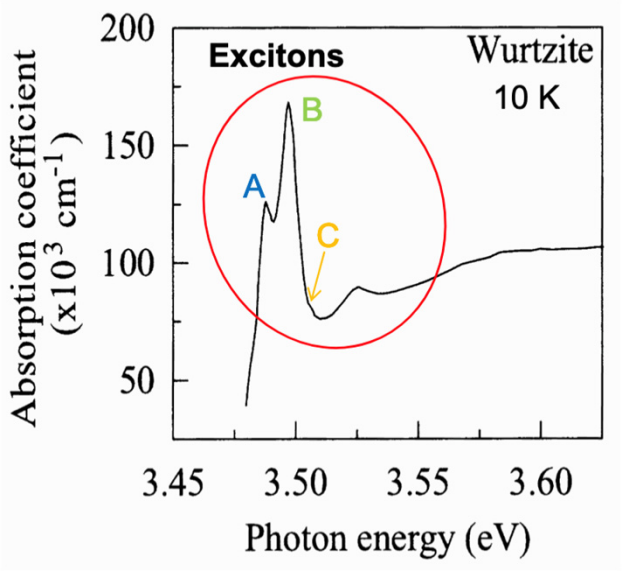
→ $$n_{op}^2(\omega) = 1 + \frac{q^2 N}{\varepsilon_0 m_0\left(\omega_0^2 - \omega^2 + i\gamma\omega\right)}$$

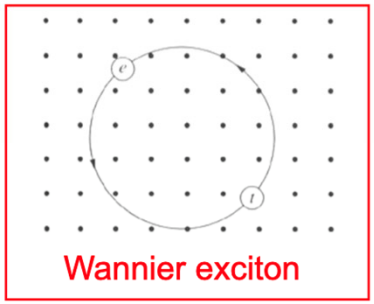
→ $$n_{op}(\lambda) = \sqrt{a + \frac{b\lambda^2}{\lambda^2 - c^2}}$$ → damping

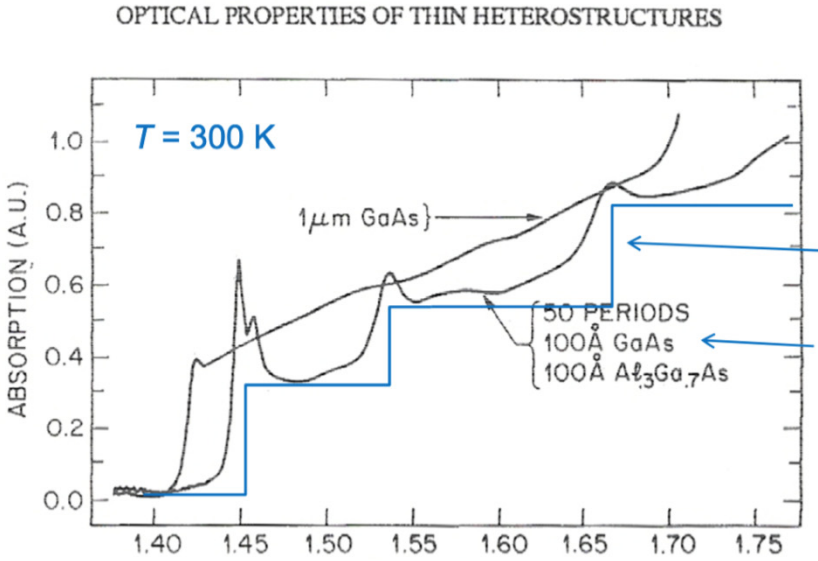
Sellmeier's law

Summary Lecture 13 - Doping and Excitons









Staircase profile

≡ 2D multiple quantum well system

Hydrogenic model

$$E_X = E_g + \frac{\hbar^2 K^2}{2M} - \frac{Ry^*}{n^2}$$

$$Ry^* = \frac{m_r}{m_0 \varepsilon_r^2} E_{H_I} = \frac{m_r e^4}{2\hbar^2 (4\pi\varepsilon_0 \varepsilon_r)^2}$$

Same formula as that used for the binding energy of impurities (but $\neq m_r$)

Ry^* = 4 meV for GaAs

Ry^* = 26 meV for GaN

Ry^* = 60-70 meV for ZnO

$$a_B = \frac{m_0}{m_r} \varepsilon_r a_0 = \frac{\hbar^2 4\pi\varepsilon_0 \varepsilon_r}{m_r e^2}$$

Bohr radius

Oscillator strength of excitons in semiconductors

Optical transition probability per unit time for converting a photon into an exciton, R_{exc}:[1]

$$R_{exc} = \frac{2\pi}{\hbar}\sum_f \left|\langle f|H_{xR}|0\rangle\right|^2 \delta\left(E_f(\mathbf{K}) - E_0 - \hbar\omega\right)$$

Fermi's Golden Rule for excitons

Exciton-photon interaction

Wavefunction accounting for the relative motion of e-h pairs (cf. hydrogenic model)

$$\langle f|H_{xR}|0\rangle = \sum_{r,k}\left(1/\sqrt{N}\right)e^{i\mathbf{k}.\mathbf{r}}\phi_{nlm}(\mathbf{r})\langle\psi_{\mathbf{k}}(\mathbf{r}_e)\psi_{-\mathbf{k}}(\mathbf{r}_h)|H_{xR}|0\rangle = \sum_{r,k}\left(1/\sqrt{N}\right)e^{i\mathbf{k}.\mathbf{r}}\phi_{nlm}(\mathbf{r})\langle\psi^c_{\mathbf{k}}|H_{eR}|\psi^v_{\mathbf{k}}\rangle$$

Number of unit cells in the crystal

Independent of **k**

with

$$\phi_{nlm}(\mathbf{r}) = R_{nl}(\mathbf{r})Y_{lm}(\theta,\phi) \quad \text{and} \quad H_{eR} = -e\mathbf{r}.\mathbf{E}$$

Electron-radiation interaction Hamiltonian in the electric dipole approximation

Associated Laguerre polynomials

Spherical harmonic

$$\sum_k e^{i\mathbf{k}.\mathbf{r}} = N\delta(\mathbf{r}) \Rightarrow \left|\langle f|H_{xR}|0\rangle\right|^2 = N\left|\phi_{nlm}(0)\right|^2\left|\langle\psi^c_{\mathbf{k}}|H_{eR}|\psi^v_{\mathbf{k}}\rangle\right|^2$$

Probability of exciting an exciton optically is $\propto$ overlap of electron-hole wavefunctions

Non-zero only for $l = 0 \Rightarrow$ only excitons with an *s* symmetry can be optically excited

[1] Yu & Cardona, Chapter 6

Oscillator strength of excitons in semiconductors

Power loss of EM field due to absorption in a unit volume of a dielectric medium:[1]

$$\text{Power loss} = R_{\text{oep}}\hbar\omega$$

Transition rate in the one-electron picture (oep)

Energy lost per unit of time

$I(z) = I_0 e^{-\alpha z}$ and $\tilde{n} = n_{\text{op}} + j\kappa$

$$-\frac{dI}{dt} = -\left(\frac{dI}{dz}\right)\left(\frac{dz}{dt}\right)$$

Phase velocity

$$\Rightarrow \frac{dI}{dz} = -\alpha I_0 e^{-\alpha z} = -\alpha I \text{ and } \frac{dz}{dt} = \frac{c}{n_{\text{op}}} \Rightarrow -\frac{dI}{dt} = \frac{c}{n_{\text{op}}}\alpha I$$

$$\alpha = \frac{4\pi\kappa}{\lambda} \Rightarrow \alpha = \frac{\varepsilon_{\text{i,oep}}\omega}{c n_{\text{op}}} \Rightarrow -\frac{dI}{dt} = \frac{\varepsilon_{\text{i,oep}}\omega I}{n_{\text{op}}^2}$$

$$R_{\text{oep}}\hbar\omega = \frac{\varepsilon_{i,\text{oep}}\omega I}{n_{\text{op}}^2} \text{ and per definition } I = \frac{n_{\text{op}}^2}{8\pi}\left|E(\omega)\right|^2$$

$$\Rightarrow \boxed{\varepsilon_{i,\text{oep}} = \frac{8\pi R_{\text{oep}}\hbar}{\left|E(\omega)\right|^2}}$$

[1] Yu & Cardona, Chapter 6

Oscillator strength of excitons in semiconductors

$$R_{\text{oep}} = \frac{2\pi}{\hbar}\sum_{\mathbf{k}_c,\mathbf{k}_v}\left|\left\langle\psi^{c}_{\mathbf{k}}\left|H_{\text{eR}}\right|\psi^{v}_{\mathbf{k}}\right\rangle\right|^2\delta\left(E_c(\mathbf{k})-E_v(\mathbf{k})-\hbar\omega\right),$$

$$\Rightarrow R_{\text{oep}} = \frac{2\pi}{\hbar}\left(\frac{e}{m_0\omega}\right)^2\left|\frac{E(\omega)}{2}\right|^2\sum_{\mathbf{k}}|P|^2\delta\left(E_c(\mathbf{k})-E_v(\mathbf{k})-\hbar\omega\right)$$

Absorption transition rate per unit volume of the crystal (See Yu&Cardona for technical details)

Electric dipole transition matrix element in the **p** representation

Imaginary part of the dielectric function for the continuum of states in the one-electron picture ($\varepsilon_{\text{i,oep}}$):[1]

$$\varepsilon_{\text{i,oep}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2\pi e}{m_0\omega}\right)^2\sum_{\mathbf{k}}|P|^2\delta\left(E_c(\mathbf{k})-E_v(\mathbf{k})-\hbar\omega\right)$$

Recommended homework:
Show using this expression of $\varepsilon_{\text{i,oep}}$ that you can recover the expression of $\alpha_0(\omega)$ obtained in slide 16 of Lecture 12

Imaginary part of the dielectric function for the exciton ($\varepsilon_{\text{i,exc}}$):[1]

$$\varepsilon_{\text{i,exc}} = \frac{2}{4\pi\varepsilon_0}\left(\frac{2\pi e}{m_0\omega}\right)^2\sum_{n=1}^{\infty}N\left|\phi_{nlm}(0)\right|^2|P|^2\delta\left(\hbar\omega-\hbar\omega_n\right) \text{ and } \left|\phi_{n0m}(0)\right|^2 = \frac{\left(na_B\right)^{-3}}{\pi}$$

Cf. slide 4

Spin degeneracy

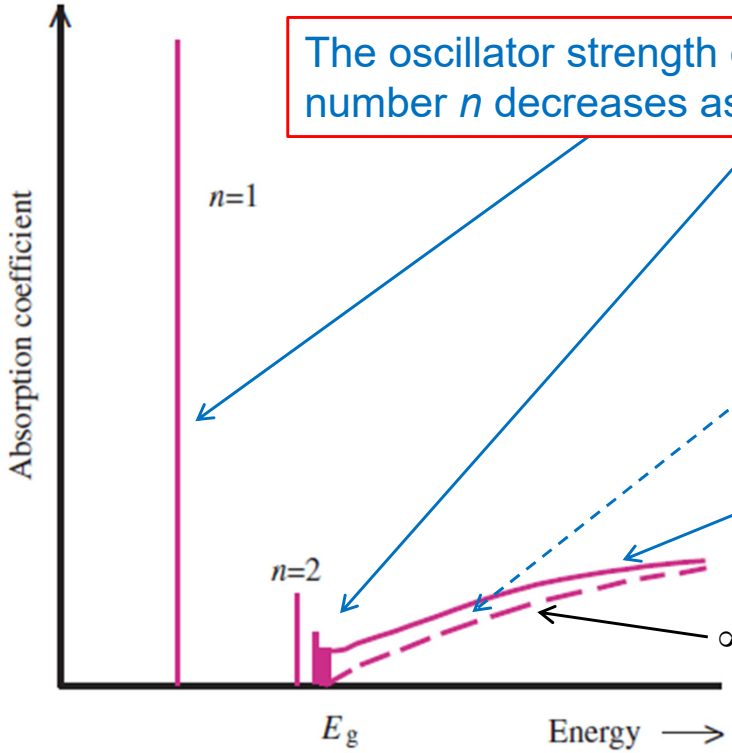
$$\varepsilon_{\text{i,exc}}(\omega) = \frac{N}{32\pi^2}\frac{e^8}{\left(m_0\omega\right)^2}\frac{m_r^3}{\hbar^7\varepsilon_0^4\varepsilon_r^3}|P|^2\sum_{n=1}^{\infty}\frac{1}{n^3}\delta\left(\omega-\omega_n\right)$$

Cf. C. Cohen-Tannoudji, B. Diu, and F. Laloë, *Quantum Mechanics*, (Wiley-VCH, Weinheim, 2020)

[1]Yu & Cardona, Chapter 6

Oscillator strength of excitons in semiconductors

Absorption spectra of a direct gap semiconductor with (solid lines) and without (broken curve) exciton effects for the continuum states[1,2]



The oscillator strength of the excitonic bound states with principal quantum number n decreases as n^{-3} while their binding energy decreases as n^{-2}

$$E_X = E_g + \frac{\hbar^2 K^2}{2M} - \frac{Ry^*}{n^2}$$

$$Ry^* = \frac{m_r}{m_0 \varepsilon_r^2} E_{H_I} = \frac{m_r e^4}{2\hbar^2 \left(4\pi\varepsilon_0 \varepsilon_r\right)^2}$$

Enhancement due to excitonic effects even in the continuum (cf. Ref. **1**): Sommerfeld enhancement factor

$\propto \sqrt{\left(E - E_g\right)}$ Cf. expression of $\alpha_0(\omega)$

[1] R. J. Elliott, Phys. Rev. **108**, 1384 (1957) (> 1900 citations)
[2] Yu & Cardona, Chapter 6

Free carrier absorption in semiconductors

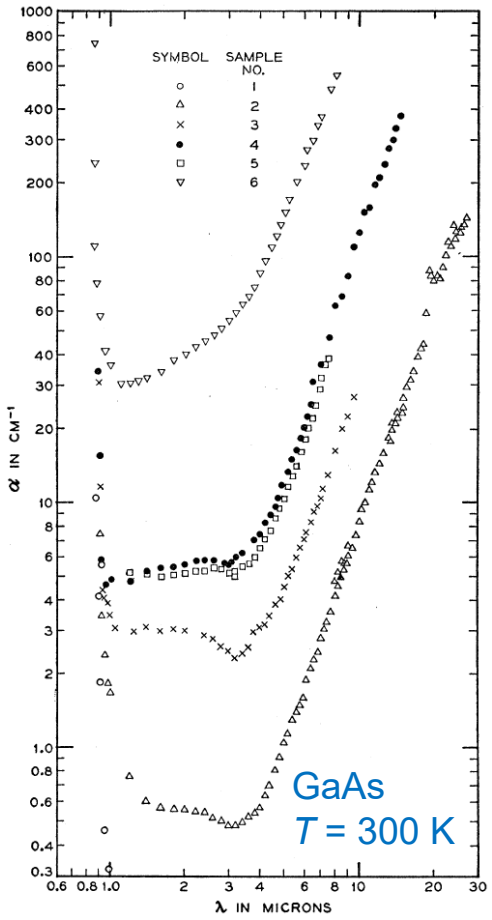


TABLE I. Electron concentration and doping impurities in GaAs samples used for optical measurements

Sample No.	Donor impurity	n (cm^{-3})
1	...	$\leq 5\times10^{14}$
2	Se	1.3×10^{17}
3	...	4.9×10^{17}
4	Se	1.09×10^{18}
5	Te	1.12×10^{18}
6	Se	5.4×10^{18}

$Z_0 \sim 377\ \Omega$ Impedance of free space

$$\alpha = \frac{\sqrt{\mu_0/\varepsilon_0}}{4\pi^2 c^2 n_{op}} \frac{n q^2 \lambda^2}{m^* \tau}$$

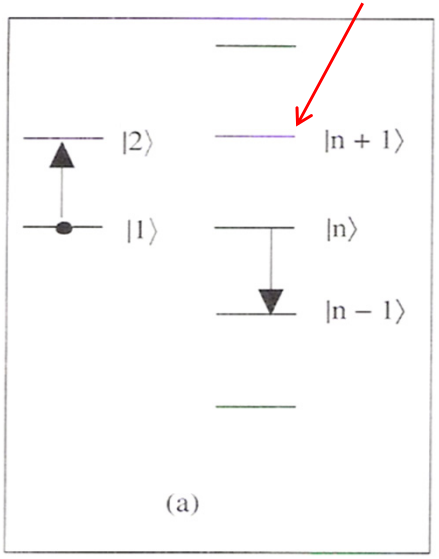
n: doping level
λ: wavelength
τ: scattering rate (10^{-14} s)

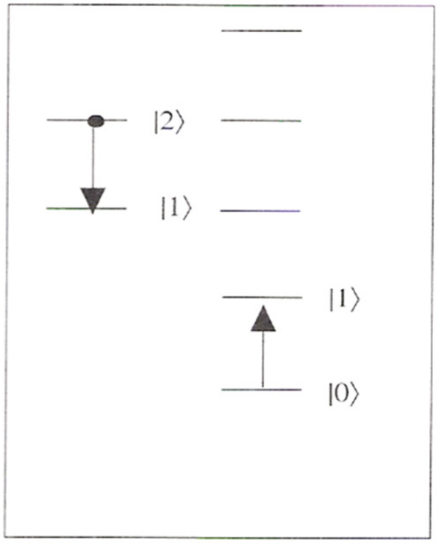
Example
Silicon doped at 10^{20} cm^{-3}
$\alpha \approx 2 \times 10^4$ cm^{-1} at 10 μm

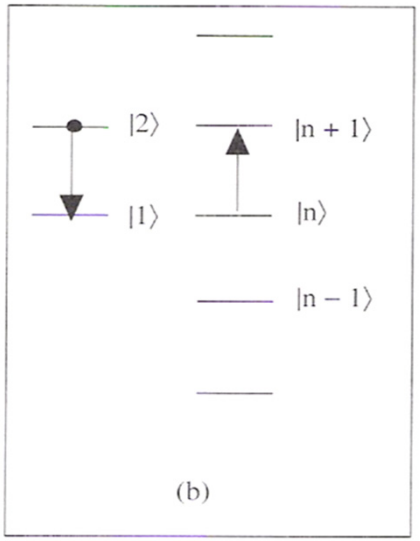
Electron-photon interaction

We consider a 2-level system with states $|i, n_l\rangle$, where i is the charged particle state (= 1 or 2 for a 2-level system) and n_l is the number of photons in mode l.

Ladder of photon number states (i.e., Fock states)







(a) Absorption **(b) Spontaneous emission** **(c) Stimulated emission**

Electron-photon interaction

Why does an electron on the excited state relax to the ground state? Or how does it release its energy?

$\Rightarrow$ **Stimulation by the vacuum field fluctuations F_l**

$$F_l = \sqrt{\frac{\hbar\omega_l}{2\varepsilon_0 L^3}} \qquad (\text{cavity } V = L^3)$$

To be admitted
More information in Chaps. 2 & 3
Rosencher-Vinter

The Hamiltonian including the electric field quantization becomes

$$\hat{W} = iqF_l(\hat{a}_l e^{i\mathbf{k}_l\mathbf{r}} - \hat{a}_l^+ e^{-i\mathbf{k}_l\mathbf{r}})\boldsymbol{\varepsilon}_\mathbf{l}\hat{r}$$

F_l *vacuum fluctuations in the mode* l

$\hat{a}_l^+, \hat{a}_l$ *creation and annihilation operators for photons in the mode* l

$\boldsymbol{\varepsilon}_\mathbf{l}$ *unit polarization vector of the electric field*

Electron-photon interaction

A few words on the vacuum field fluctuations

- Hamiltonian of mode l of an EM wave $H_{\perp l} = \hbar\omega_l\left(\hat{a}_l^+\hat{a}_l + \frac{1}{2}\right)$

  Photon number operator in mode l

  Normal component of the Hamiltonian, which is the only one to be of interest (Sec. 2.3 Rosencher-Vinter)

- Stationary state energy $E_{l,i} = \hbar\omega_l\left(i_l + \frac{1}{2}\right)$

  Number of photons in mode l

- Vacuum energy of mode l (state $|0_l\rangle$) (inherited from Heisenberg uncertainty principle) $E_{vacuum,l} = \frac{1}{2}\hbar\omega_l$

Mean value and variance of the electric field

$$\overline{\left(\hat{\varepsilon}_{\perp l}\right)} = \langle i_l|\hat{\varepsilon}_{\perp l}|i_l\rangle = 0$$

$$\overline{\left(\hat{\varepsilon}_{\perp l}\right)^2} = \langle i_l|\left(\hat{\varepsilon}_{\perp l}\right)^2|i_l\rangle = F_l^2\left(2i_l+1\right)$$

with $\hat{\varepsilon}_{\perp l} = iF_l(\hat{a}_l e^{i\mathbf{k}_l\mathbf{r}} - \hat{a}_l^+ e^{-i\mathbf{k}_l\mathbf{r}})\boldsymbol{\varepsilon}_\mathbf{l}$

See sections 2.5 and 2.6 in the book by Rosencher & Vinter

Even for an empty cavity (i_l= 0), the variance of the electric field is nonzero and proportional to the square of the *vacuum field fluctuations*

Experimental signature: **Lamb shift** (degeneracy lift between $2s_{1/2}$ and $2p_{1/2}$ states in the hydrogen atom) signature of the coupling of H atoms with vacuum field fluctuations (full theory: *QED theory Feynman-Schwinger-Tomonaga, Nobel Prize 1965*)

Electron-photon interaction

Let us consider the absorption rate of a 2-level system experiencing an excitation with *n* photons within a mode *l*

The initial state is $|1,n\rangle$ and the final state is $|2,n\text{-}1\rangle$: *there is 1 photon less*

$$P_{1,2} = \frac{2\pi}{\hbar}\left|\langle 2,n-1|\hat{W}|1,n\rangle\right|^2 \delta\left(\hbar\omega = E_2 - E_1\right)$$ **Fermi's Golden Rule**

with the field quantization of the perturbation $\hat{W} = iqF_l(\hat{a}_l e^{i\mathbf{k}_l\mathbf{r}} - \hat{a}_l^+ e^{-i\mathbf{k}_l\mathbf{r}})\boldsymbol{\varepsilon}_l\hat{\mathbf{r}}$ $\Rightarrow$

$$P_{1,2} = \frac{2\pi}{\hbar}q^2F_l^2\left|\langle 2,n-1|\left(\hat{a}_l e^{i\mathbf{k}_l\mathbf{r}} - \hat{a}_l^+ e^{-i\mathbf{k}_l\mathbf{r}}\right)\boldsymbol{\varepsilon}_l\hat{\mathbf{r}}|1,n\rangle\right|^2 \delta\left(\hbar\omega = E_2 - E_1\right)$$

The electron state is independent of the photon state (i.e., we have two separable Hilbert spaces)

$\Rightarrow$

$$P_{1,2} = \frac{2\pi}{\hbar}q^2F_l^2\underbrace{\left|\langle n-1|\left(\hat{a}_l e^{i\mathbf{k}_l\mathbf{r}} - \hat{a}_l^+ e^{-i\mathbf{k}_l\mathbf{r}}\right)|n\rangle\right|^2}_{\langle n-1|\hat{a}_l|n\rangle = \sqrt{n}}\left|\langle 2|\boldsymbol{\varepsilon}_l\hat{\mathbf{r}}|1\rangle\right|^2 \delta\left(\hbar\omega = E_2 - E_1\right)$$

only non-zero related term

Electron-photon interaction

The absorption rate is then given by

$$P_{1,2} = \frac{2\pi}{\hbar} q^2 F_l^2 n \left|\langle 2|\boldsymbol{\varepsilon}_l \hat{\mathbf{r}}|1\rangle\right|^2 \delta(\hbar\omega = E_2 - E_1)$$

$$F_l = \sqrt{\frac{\hbar\omega_l}{2\varepsilon_0 L^3}}$$

$$P_{1,2} = \frac{2\pi}{\hbar} q^2 \frac{\hbar\omega_l}{2\varepsilon_0 L^3} n \left|\langle 2|\boldsymbol{\varepsilon}_l \hat{\mathbf{r}}|1\rangle\right|^2 \delta(\hbar\omega = E_2 - E_1)$$

On the other hand, the amplitude of the classical electric field is such that $E_0 = \sqrt{\frac{2\hbar\omega_l}{\varepsilon_0 L^3} n}$

Thus, $P_{1,2} = \frac{\pi}{2\hbar} q^2 E_0^2 \left|\langle 2|\boldsymbol{\varepsilon}_l \hat{\mathbf{r}}|1\rangle\right|^2 \delta(\hbar\omega = E_2 - E_1)$ Cf. Lecture 12, slides 5-8

Absorption rate of a photon (with *n* photons in mode *l*) with an electron transition from state 1 to state 2

Electron-photon interaction

The recombination rate is calculated in the same way

The initial state is $|2,n\rangle$ and the final state is $|1,n+1\rangle$: *1 photon is created*

$$P_{2,1} = \frac{2\pi}{\hbar} q^2 F_l^2 \underbrace{\left|\left\langle n+1\left|\left(\hat{a}_l e^{i\mathbf{k}_l\mathbf{r}} - \hat{a}_l^+ e^{-i\mathbf{k}_l\mathbf{r}}\right)\right|n\right\rangle\right|^2}_{\left\langle n+1\left|\hat{a}_l^+\right|n\right\rangle = \sqrt{n+1}} \left|\left\langle 2\left|\boldsymbol{\varepsilon}_\mathbf{l}\hat{\mathbf{r}}\right|1\right\rangle\right|^2 \delta\left(\hbar\omega = E_2 - E_1\right)$$

only non-zero related term

Thus, we have

$$P_{2,1} = \frac{2\pi}{\hbar} q^2 \frac{\hbar\omega_l}{2\varepsilon_0 L^3}(n+1)\left|\left\langle 2\left|\boldsymbol{\varepsilon}_\mathbf{l}\hat{\mathbf{r}}\right|1\right\rangle\right|^2 \delta\left(\hbar\omega = E_2 - E_1\right)$$

Recombination rate of an electron from state 2 to state 1 with 1 photon emitted in mode *l*

Electron-photon interaction

Recombination rate in a two-level system

$$P_{2,1} = q^2 \frac{\pi\omega_l}{\varepsilon_0 L^3} n \left|\langle 2|\boldsymbol{\varepsilon}_l \hat{\mathbf{r}}|1\rangle\right|^2 \delta(\hbar\omega = E_2 - E_1) + q^2 \frac{\pi\omega_l}{\varepsilon_0 L^3} \left|\langle 2|\boldsymbol{\varepsilon}_l \hat{\mathbf{r}}|1\rangle\right|^2 \delta(\hbar\omega = E_2 - E_1)$$

Stimulated emission — **Spontaneous emission (P_{sp})**

Stimulated emission $\Rightarrow$ proportional to the photon number n

Spontaneous emission $\Rightarrow$ due to vacuum fluctuations F_l

The **spontaneous emission rate** is calculated over all the cavity modes

$$\Gamma_{sp} = \iiint P_{sp} \mathrm{d}^3\mathbf{k}$$

+ substitute $\varepsilon_0 \rightarrow \varepsilon_r \varepsilon_0$

Section 3.6 Rosencher-Vinter

$$\Gamma_{sp} = \frac{q^2 r_{12}^2 \omega^3 n_{op}}{3\pi c^3 \hbar \varepsilon_0} = 1/\tau_{sp}$$

τ_{sp} *radiative lifetime*

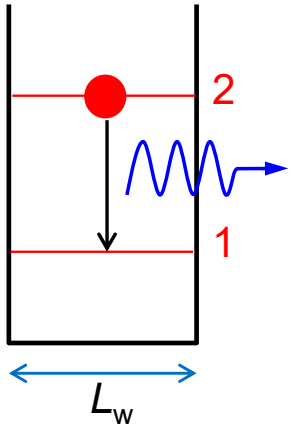
First derivation: Weisskopf-Wigner theory 1930

V. Weisskopf and E. Wigner, Z. Phys. **63**, 54 (1930) (> 1500 citations)

Spontaneous emission: case of discrete levels

Application:

Recombination lifetime in a quantum well with infinite barriers



1) Energy level

$$E_n = n^2 \frac{\hbar^2 \pi^2}{2m^* L_w^2}$$

Here, *n* is the principal quantum number of the quantized energy levels and not the photon number in the optical mode!

Then,

$$\hbar\omega_{12} = E_2 - E_1 = 3\frac{\hbar^2 \pi^2}{2m^* L_w^2}$$

2) Dipole element r_{12}

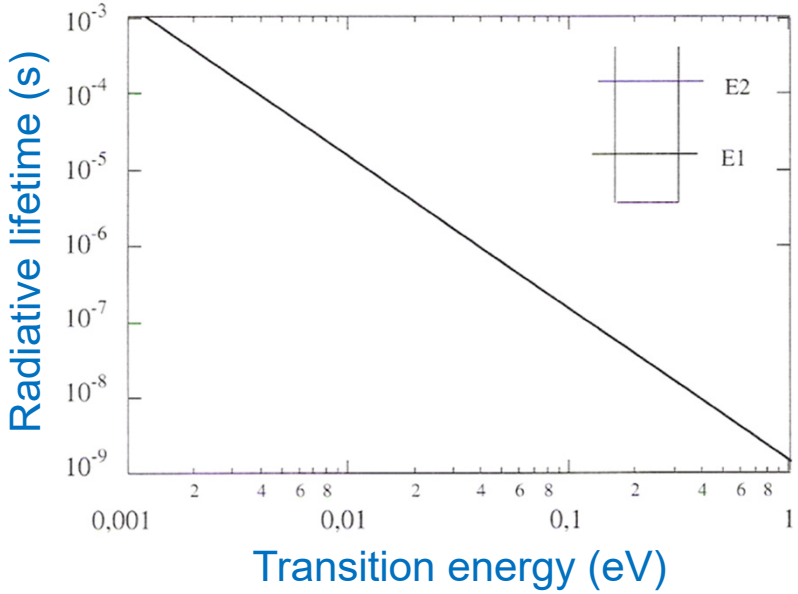
$$r_{12} = \langle \Psi_2(x) | x | \Psi_1(x) \rangle \quad \text{with} \quad \Psi_1(x) = \sqrt{\frac{2}{L_w}} \cos(\pi x / L_w) \quad \text{and} \quad \Psi_2(x) = \sqrt{\frac{2}{L_w}} \sin(2\pi x / L_w)$$

Finally, $r_{12} = \frac{2^4 L_w}{3^2 \pi^2}$ and $r_{12}^2 = \frac{2^7 m^*}{3^3 \pi^2 \hbar^2} 1/E_{12}$

Spontaneous emission: case of discrete levels

Radiative lifetime: $$1/\tau_{sp} = \frac{q^2 r_{12}^2 \omega^3 n_{op}}{3\pi c^3 \hbar \varepsilon_0}$$

$$\tau_{sp} = \frac{3^4 \pi^3 m^* c^3 \hbar^2 \varepsilon_0}{2^7 q^2 n_{op}} \frac{1}{E_{12}^2}$$



Transition energies > 1 eV
⇒ radiative lifetime ~1 ns

Values to be also compared to the optical phonon lifetime (slide #27 of Lecture 6) for a rough comparison !

Spontaneous emission in a bulk system

In an intrinsic bulk semiconductor:

The spontaneous recombination rate $r_{sp}(\mathbf{k})$ (s^{-1}) between the CB and the VB is given for a state with a wavevector **k**

$$r_{sp}(\mathbf{k}) = A_{cv}\, f_c(E_c)\,(1\text{-}f_v(E_v))$$

with $A_{CV} = 1/\tau_R$ *the spontaneous recombination rate*

and the radiative lifetime is given by

$$\tau_R = \frac{\pi c^3 \hbar \varepsilon_0}{q^2 x_{vc}^2 n_{op} \omega_{vc}^3} = \frac{2\pi c^3 \hbar^2 \varepsilon_0 m_0}{q^2 n_{op} E_g E_p}$$

$\tau_R \uparrow$ *when* $E_g \downarrow$ because, overall, the $n_{op}E_gE_p$ product decreases!

$\Rightarrow$ *It is more challenging to achieve a laser based on a wide bandgap SC!*

Spontaneous emission in a bulk system

In an intrinsic bulk semiconductor:

The spectral distribution of spontaneous recombination rate $R_{sp}(h\nu)$ due to a quasi-equilibrium distribution of carriers is then given by

$$R_{sp}(h\nu) = 2\sum_{\mathbf{k}} r_{sp}(\mathbf{k}) = 2\sum_{\mathbf{k}} \frac{1}{\tau_R(\mathbf{k})} f_c(\mathbf{k})\left(1 - f_v(\mathbf{k})\right)\delta\left(E_c - E_v = h\nu\right)$$

The summation is performed over all the **k**-vectors verifying the energy conservation condition (hence the Dirac delta)

$$E_c(\mathbf{k}) - E_v(\mathbf{k}) = h\nu = E_g + \frac{\hbar^2 k^2}{2m_r}$$

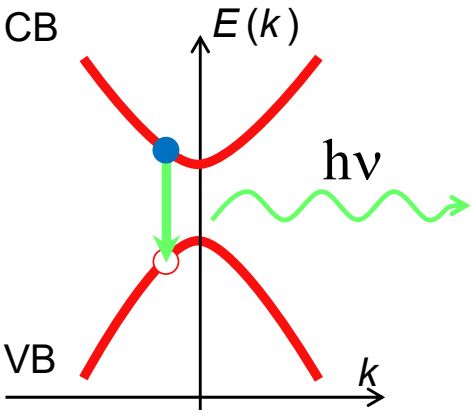
which leads to

$$R_{sp}(h\nu) = \int_0^\infty r_{sp}(E)\rho_j(E)\delta(E = h\nu)\,dE = r_{sp}(h\nu)\rho_j(h\nu)$$ using the fact that $2\sum_{\mathbf{k}} \leftrightarrow \int_{\mathbf{k}} \rho(\mathbf{k}) d^3\mathbf{k} \leftrightarrow \int_E \rho(E) dE$

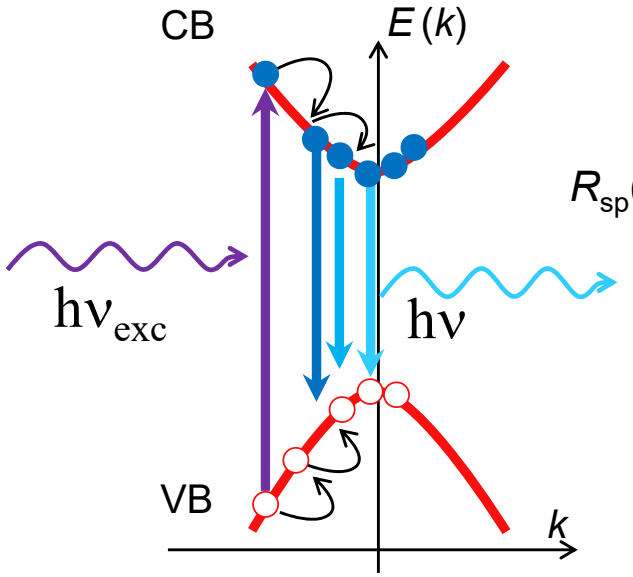
$$\boxed{R_{sp}(h\nu) = \frac{1}{\tau_R}\rho_j(h\nu) f_c\left(E_c(h\nu)\right)\left(1 - f_v\left(E_v(h\nu)\right)\right)}$$

Spontaneous emission in a bulk system

Recombination and spontaneous emission

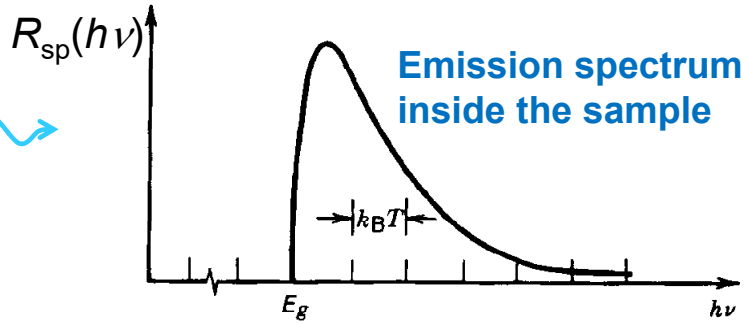


Spontaneous emission



Photoluminescence
(Electroluminescence)
(Cathodoluminescence)

van Roosbroeck-Shockley relation: link between emission rate and absorption at thermal (or quasi-thermal) equilibrium



Emission spectrum

$$R_{sp}(h\nu) = r_{sp}(h\nu)\,\rho_j(h\nu)$$

$$R_{sp}(h\nu) = \alpha(h\nu)\,\frac{8\pi n_{op}^2 (h\nu)^2}{h^3 c^2}\,\frac{1}{e^{\frac{h\nu}{k_B T}} - 1}$$

Planck radiation law

Spectral distribution of spontaneous recombination rate

W. van Roosbroeck and W. Shockley, Phys. Rev. **94**, 1558 (1954) (> 900 citations)

Bimolecular recombination coefficient

Total radiative recombination rate

$$R_{sp} = \frac{1}{\tau_R}\int_{E_g}^{\infty} \rho_j(h\nu)\, f_c\left(E_c(h\nu)\right)\left(1 - f_v\left(E_v(h\nu)\right)\right) \mathrm{d}h\nu$$

i.e., $$R_{sp} = \frac{e^{\left(E_{Fc} - E_{Fv}\right)/k_B T}}{\tau_R}\int_{E_g}^{\infty} \rho_j(h\nu)\, e^{-h\nu/k_B T}\, \mathrm{d}h\nu$$

Expression valid within Boltzmann approximation!

Can be recast into a constant × the gamma function (aka the *Euler integral of the 2nd kind*)!

which leads to

$$R_{sp} = \frac{1}{\tau_R} N_j e^{\left(E_{Fc} - E_{Fv} - E_g\right)/k_B T}$$ with N_j the effective density of states with reduced mass m_r

Finally, we get: $$R_{sp} = \frac{1}{\tau_R}\frac{N_j}{N_c N_v} np$$

$$R_{sp} = Bnp$$

with

$$B = \frac{1}{\tau_R}\frac{N_j}{N_c N_v} = \frac{1}{\tau_R N_c}\left(\frac{m_r}{m_v}\right)^{3/2}$$

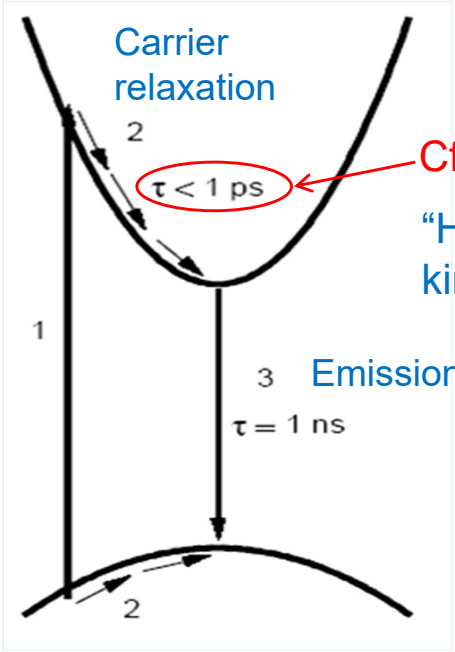
Cf. Lecture 7
Band-to-band recombinations
Slides 18-21

Material	B (cm^3 s^{-1})
GaAs	7.2×10^{-10}
GaSb	2.4×10^{-10}
InP	1.3×10^{-9}
InAs	8.5×10^{-11}
InSb	4.6×10^{-11}

@ 300 K

Spontaneous emission

Photoluminescence

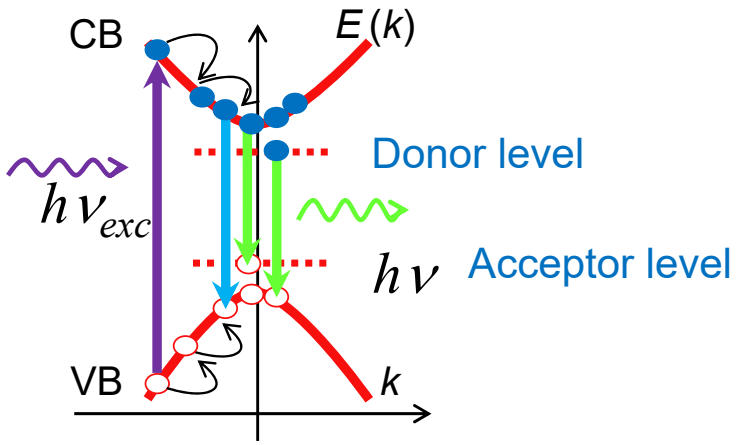


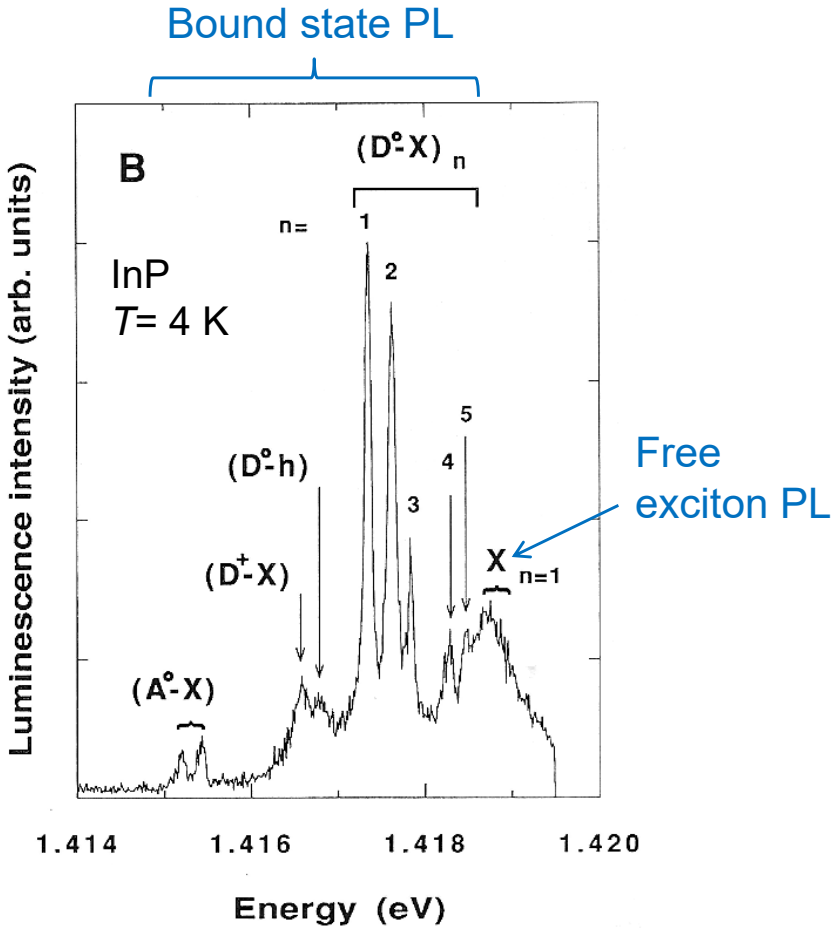
Cf. T_2 relaxation time in the expression of χ

"Hot" carriers (electrons and holes) release their kinetic energy *via* LO and acoustic phonon emission

Spontaneous emission

Photoluminescence (case of bulk InP)

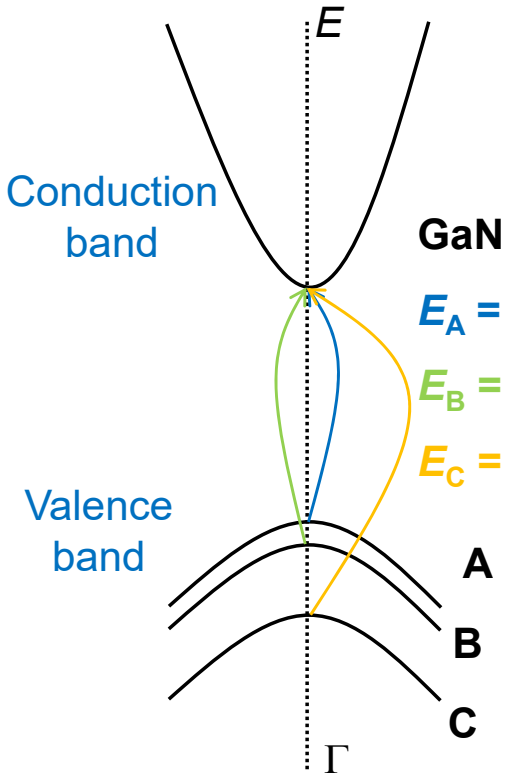




⇒ *How can we discriminate intrinsic PL from extrinsic one?*

Spontaneous emission

Photoluminescence and reflectivity (GaN)



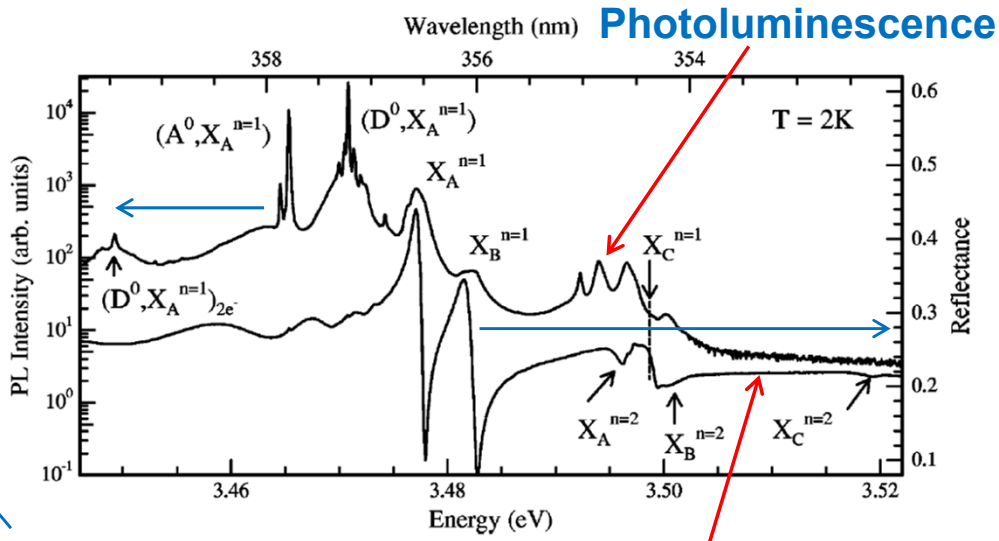
GaN (10 K):

E_A = 3.478 eV

E_B = 3.485 eV

E_C = 3.503 eV

Expected values in relaxed (strain-free) bulk GaN



Reflectivity (∝ JDOS, i.e., *A* or f_{osc})

$R \cong 1\text{-}A$

Exciton binding energy can also be determined from reflectivity spectra!

Examination protocol

Examination: 23/01/2025 from 3:15pm until 6:15pm in room BC 0 1

- The exam could cover any of the topics addressed during the 14 lectures of this semester and the corresponding series.
- The exam will be a mixture of problem solving (involving basic mathematical calculations) and analysis of figures with a main focus on the underlying physics. Special care will be paid to the quality and the correctness of your explanations and/or arguments.
- The exam shall be written in *readable* English (**pay a special attention to your handwriting…**).
- Printed versions of the lectures with your personal notes as well as the exercises (+ solutions) are allowed (+ summary of the core concepts posted on Moodle) but you cannot bring anything else (no textbook, etc.)! **Any tool providing access to the Internet is strictly forbidden (no smartphone and smartwatch).**
- The reference for all the courses of this semester will be the version of the lectures posted on the Moodle repository, i.e., the files available from December 20 2024!
- Do not forget to bring a scientific calculator for numerical applications. **There won't be any of them on loan!**
- You can reach me preferably by e-mail (from January 3 2025): raphael.butte@epfl.ch (Samuele will always forward your requests to me so please use the shortest pathway!)